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IMPROVED BERNOULLI SUB-EQUATION FUNCTION METHOD FOR EXACT SOLUTIONS OF CONFORMABLE TIME FRACTIONAL RLW EQUATION

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In this paper, we consider conformable time fractional Regularized Long Wave (RLW) equation with the form

$$D_t^\alpha u + pu_x + quu_x + rD_t^\alpha u_{xx} = 0, t > 0, \tag{1}$$

where p, q and r real parameters, D_t^α is the conformable fractional differential operator and $u = u(x, t)$. In this study, we obtain the exact solutions of (1) using Improved Bernoulli Sub-Equation Function Method (IBSEFM) and give the 3D graphs acquired from the values of the solutions.

Keywords: conformable time fractional regularized long wave equation, IBSEF method.

Introduction

In recent years,the fractional differential equations have become a useful tool for describing nonlinear phenomena of science and engineering models.Many of techniques applied to nonlinear partial differential equations have been adapted for fractional nonlinear partial differential equations to find exact solutions. For example the functional variable method [1], the first integral method [2], the exp-function method [3] and many others.In [4] a new simple well behaved definition of the fractional derivative called conformable fractional derivative is introduced. The conformable fractional derivative is theoretically easier than fractional derivative to handle. Also the conformable fractional derivative obeys some conventional properties that can't be satisfied by the existing fractional derivatives, for instance; the chain rule [5]. The conformable fractional derivative has the weakness that the fractional derivative of any differentiable function at the point zero is equal to zero. So that in [6-8] it is proposed a suitable fractional derivative that allows us to escape the lack of the conformable fractional derivative.

1. Conformable Fractional Derivative

In this section, we give some basic definition, properties and theorems about the conformable fractional derivative.

The conformable derivative of order α with respect to the independent variable t is defined as [9]

$$D_t^a(y(t)) = \lim_{\tau\to 0}\frac{y(t+\tau t^{1-\alpha}) - y(t)}{\tau},\ t>0,\ \alpha\in(0,1]$$

for a function $y = y(t) : [0,\infty) \to \mathbb{R}$.

Теорема 1. *Assume that the order of the derivative $\alpha \in (0,1]$ and suppose that $u = u(t)$ and $y = y(t)$ are $\alpha-$ differentiable for all positive t. Then,*

1. $D_t^a(c_1u + c_2y) = c_1D_t^a(u) + c_2D_t^a(y)$.

2. $D_t^a(t^k) = kt^{k-x}, \forall\ k \in \mathbb{R}$.

3. $D_t^a(\lambda) = 0$, *for all constant function* $u(t) = \lambda$.

4. $D_t^a\ (uy) = uD_t^a(y) + yD_t^a(u)$.

5.$D_t^a\left(\frac{u}{y}\right) = \frac{yD_t^a(u) - uD_t^a(y)}{y^2}$

6.$D_t^a\ (u)\ (t) = t^{1-\alpha}\frac{du}{dt}$ *for* $\forall\ c_1, c_2 \in \mathbb{R}$.

Conformable fractional differential operator satisfies some critical fundamental properties like the chain rule, Taylor series expansion and Laplace transform.

Теорема 2. *Let $u = u(t)$ be an $\alpha-$ conformable differentiable function and assume that y is differentiable and defined in the range of u. Then,*

$$D_t^a(u\circ y)(t) = t^{1-\alpha}y'(t)u'(y(t)).$$

The proofs of these theorems are given in [4] and in [7] respectively.

2. Basic Properties of The Improved Bernoulli Sub-Equation Function Method(IBSEFM)

In this section we will give the fundamental properties of the Improved Bernoulli sub-equation function method (IBSEFM) formed by modifying the Bernoulli sub-equation function method [10]. We consider the following four steps:

Step 1: Let us consider the following conformable time-fractional PDE of the form

$$P(u, D_t^\alpha u, u_x, D_{tt}^{2\alpha}u, u_{xx}, ...) = 0, \tag{2}$$

and take the wave transformation;

$$u(x,t) = U\left(\eta\right),\quad \eta = \left(x - \frac{vt^\alpha}{\alpha}\right), \tag{3}$$

where v is a constant to be determined later. Using chain rule and substituting (3) in (2) we obtain the following nonlinear ordinary differential equation;

$$N(U, U', U'', ...) = 0. \tag{4}$$

Step 2: Considering trial equation of solution in (4) it can be written as following;

$$U(\eta)=\frac{\sum\limits_{i=0}^{n}a_iF^i(\eta)}{\sum\limits_{j=0}^{m}b_jF^j(\eta)}=\frac{a_0+a_1F(\eta)+a_2F^2(\eta)+...a_nF^n(\eta)}{b_0+b_1F(\eta)+b_2F^2(\eta)+...b_mF^m(\eta)} \tag{5}$$

According to the Bernoulli theory, we can consider the general form of Bernoulli differential equation for as following;

$$F'(\eta)=\sigma F(\eta)+dF^M(\eta),\ \sigma\neq 0, d\neq 0,\ M\in\mathbb{R}-\{0,1,2\}, \tag{6}$$

where $F(\eta)$ is Bernoulli differential polynomial. Substituting (5) and (6) in (4) it yields us an equation of polynomial $\Omega(F)$ of F as following;

$$\Omega(F(\eta))=\rho_sF(\eta)^s+...+\rho_1F(\eta)+\rho_0=0.$$

According to the balance principle, we can determine the relationship between n,m and M.

Step 3: The coefficients of $\Omega(F(\eta))$ all be zero will give us an algebric system of equations;

$$\rho_i=0,\quad i=0,...,s.$$

Solving this system, we will specify the values of $a_0,a_1,...,a_n$ and $b_0,b_1,...,b_m$.

Step 4: When we solve differential equation (6), we obtain the following two situations according to σ and d,

$$F(\eta)=\left[\frac{-d}{\sigma}+\frac{E}{e^{\sigma(M-1)\eta}}\right]^{\frac{1}{1-M}},\quad \sigma\neq d, \tag{7}$$

$$F(\eta)=\left[\frac{(E-1)+(E+1)\tanh(\sigma(1-M))\frac{\eta}{2}}{1-\tanh\left(\sigma(1-M)\frac{\eta}{2}\right)}\right], \sigma=d,\ E\in\mathbb{R}.$$

Using a complete discrimination system for polynomial of $F(\eta)$, we obtain the analytical solutions of (4) with the help of software programme and classify the exact solutions of (4). For a better interpretations of obtained results, we can plot two and three dimensional figures of analytical solutions by considering suitable values of parameters.

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