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ON THE EXACT SOLUTION OF FRACTIONAL SIMPLIFIED MCH EQUATION

U. Demirbilek,V. Ala, Kh. R. Mamedov, S. Goktas
(Mersin, TURKEY)
ulviyedemirbilek@gmail.com, volkanala@mersin.edu.tr,
hanlar@mersin.edu.tr, srtcgoktas@gmail.com

In this work, we consider the simplified modified Camassa-Holm (MCH) equation by applying the new extended direct algebraic method. The simplified MCH equation is given as follows:

$$D_t^{(\mu)} + \rho D_x^{(\mu)} - u_{xxt}^{(3\mu)} + p(u^3)_x^{(\mu)} = 0, \quad t \geq 0, \quad 0 < \mu \leq 1, \tag{1}$$

where p, ρ real parameters, D_t^{μ} is the conformable derivate operator of the order μ respect to the variable. In this study,the exact solutions of (1) using Improved Bernoulli Sub-Equation Function Method (IBSEFM) are obtained and the 3D graphs acquired from the values of the solutions are given.

Keywords: IBSEF method, Conformable fractional derivative, Camassa–Holm equation.

Introduction

The present work is devoted to solving the simplified modified Camassa–Holm (MCH) equation by applying the Improved Bernoulli sub-equation function method. Several methods have been implemented to solve the simplified MCH equation by many authors [1–7] , such as sine-Gordon expansion method [1], the (G'/G)-expansion method and the generalized Riccati equation [2], modified extended tanh method [3].

The definition of conformable fractional derivative of order $\mu \in (0, 1]$, it is defined by the following expression [6]:

$$\frac{d^{\mu} f(t)}{dt^{\mu}} = \lim_{\varepsilon \to 0} \frac{f\left(t + \varepsilon t^{1-\mu}\right) - f(t)}{\varepsilon}; \ f : (0, \infty] \to \mathbb{R}.$$

Also, conformable fractional derivative has the following main properties [6, 7]:

i. $\frac{d^{\mu} t^{\vartheta}}{dt^{\mu}} = \vartheta t^{\vartheta - \mu}, \quad \forall \mu \in \mathbb{R},$

ii. $\frac{d^{\mu} fg}{dt^{\mu}} = f \frac{d^{\mu} g}{dt^{\mu}} g + g \frac{d^{\mu} f}{dt^{\mu}},$

iii. $\frac{d^{\mu}(f \circ g)}{dt^{\mu}} = t^{1-\mu} g'(t) f'(g(t)).$

In this study, we will reach the exact solution of our equation by means of software by using the basic features given in [6, 7] and the method given in [8].

Solutions of fractional simplified MCH equation

Consider a nonlinear conformable fractional partial differential equation of the form

$$P\left(u, u_t^{(\mu)}, u_x^{(\mu)}, u_{xt}^{(2\mu)}, u_{xxt}^{(3\mu)}, ...\right) = 0, \tag{2}$$

where U is an unknown function, P is a polynomial of U and their derivatives and partial conformable fractional derivatives. Using the wave transformation, as:

$$u\left(x,t\right) = U\left(\xi\right), \quad \xi = \frac{x^\mu}{\mu} - r\frac{t^\mu}{\mu}, \tag{3}$$

where r is an arbitrary constant.

We use Eq. (3) to reduce Eq. (1) to the following nonlinear ordinary differential equation

$$rU'' + (\rho - r)U + sU^3 = 0. \tag{4}$$

When we apply the balance between U'' and U^3, we obtain following relationship for m, n and M :

$$M = n - m + 1.$$

Family 1. According to the balance principle, if we take $M = 3$, $n = 3$, and $m = 1$, we can write

$$U(\xi) = \frac{\sum\limits_{i=0}^{n} a_i F^i(\xi)}{\sum\limits_{j=0}^{m} b_j F^j(\xi)} = \frac{a_0 + a_1 F(\xi) + a_2 F^2(\xi) + a_3 F^3(\xi)}{b_0 + b_1 F(\xi)} = \frac{\psi}{\phi}, \tag{5}$$

$$U'(\xi) = \frac{\psi'\phi - \phi'\psi}{\phi^2},$$

$$\vdots,$$

where $F(\xi)$ is Bernoulli differential polynomial, $F'(\xi) = \sigma F(\xi) + dF^M(\xi)$, $\sigma \neq 0, d \neq, a_3 \neq 0, b_1 \neq 0,\ M \in \mathbb{R} - \{0,1,2\}$. When we use Equations (5) in Equation (4), we obtain a system of algebraic equations from the coefficients of polynomial of F. By solving this algebraic system of equations with the help of software programme, it yields us the following coefficients:

case1. For $\sigma \neq d$,

$$a_0 = -\frac{i\sqrt{-p+r}a_2}{2\sqrt{2}d\sqrt{r}}; a_1 = -\frac{i\sqrt{-p+r}a_2 b_1}{2\sqrt{2}d\sqrt{r}b_0}; a_3 = -\frac{a_2 b_1}{b_0}; s = -\frac{8d^2 r b_0^2}{a_2^2}; \quad (6)$$

$\sigma = -\frac{i\sqrt{-p+r}}{\sqrt{2}\sqrt{r}}$.

Putting Equation (6) coefficients in Equation (5), we obtain the complex exponential function solution to the conformable time MCH equation as follows:

$$U_1(x,\ t) = \frac{\left(\frac{-i\sqrt{2}\sqrt{-p+r}}{d\sqrt{r}} + \frac{4}{\frac{-i\sqrt{2}d\sqrt{r}}{\sqrt{-p+r}} + exp\frac{-i\sqrt{2}\sqrt{-p+r}(-ct^{\alpha}x^{\alpha})}{\sqrt{r}\alpha}\epsilon} \right)\alpha_2}{4b_0}, \quad (7)$$

where a_2, b_0, α, d, r are constants and not zero.

As a result, we have succesfully applied the IBSEFM to the nonlinear conformable time fractional MCH equation and we obtained the complex prototype solution.Using software mathematics programme, it is constructed two and three dimensional figures of this solution according to the suitable values of these parameters.

REFERENCES

[1] *Bulut H., Sulaiman T. A., Erdogan F., Baskonus H. M.* On the new hyperbolic and trigonometric structures to the simplified MCH and SRLW equations // EurPhys J Plus, 2017, 132:350. Eur. Phys. J. Plus. 2017. Vol. 132, Article no. 350. DOI: https://doi.org/10.1140/epjp/i2017-11619-1

[2] *Zaman M. M., Sayeda S.*, Traveling Wave Solutions for the Nonlinear Evolution Equation via the Generalized Riccati Equation and the (G'/G)-Expansion Method // World Applied Sciences Journal. 2013. Vol. 22, № 3. P. 396–407.

[3] *Khalid K. Ali, Nuruddeen R. I., Raslan K. R.* New structures for the space-time fractional simplified MCH and SRLW equations // Chaos. Solitons and Fractals. 2018. Vol. 106. P. 304–309. DOI: https://doi.org/10.1016/j.chaos.2017.11.038

[4] *Lu B.* The first integral method for some time fractional diferential equations // J. Math. Anal. Appl. 2012. Vol. 395, № 2. P. 684–693.

[5] *Bin Z.* 4 // The Sci. World J. 2013. Vol. 2013. P. 1–8.

[6] *Khalil R., Al Horani M., Yousef A., Sababheh M.* A new definition of fractional derivative // Pramana. 2013. Vol. 81, № 3. P. 377–384.

[7] *Eslami M., Rezazadeh H.,* The first integral method for Wu Zhang system with conformable time-fractional derivative // Calcolo. 2016. Vol. 53 № 3. P. 475–485.

[8] *Bulut H., Yel G., Baskonus H. M.* An Application of Improved Bernoulli Sub-Equation Function Method to The Nonlinear Time-Fractional Burgers Equation // Turk. J. Math. Comput. Sci. 2016. № 5. P. 1–7.